1. True-False Questions

1	T	There exists at least one DFA for any regular language.
2	T	There exists at least one CFG for any regular language.
3	F	There cannot be any CFG for a non-regular language.
4	T	If the language is regular, then we can construct a PDA for it.
5	F	$\{1^n0^n \| n>=0\}$ is a regular language.
6	F	$\{0^n1^n \| n>0\}$ is a regular language.
7	T	If there is a CFG for a language, then we can always construct a PDA for it.
8	F	If there is a CFG for a language, then we can always construct a NFA for it.
9	T	The power of DFA, NFA and Regular Expressions are equal.
10	T	Turing Machine is more powerful then PDA.
11	F	A non-regular is always a CFG.
12	F	All non-regular languages are recognized by PDA.
13	T	There is a stack with an infinite size for PDAs
14	F	NFA is more powerful than DFA
15	T	Pumping Lemma is often used for proving that a particular language is non-regular (xyz)
16	T	Pumping Lemma is often used for proving that a particular language is non-CFG (uvxyz)
17	T	A PDA can be constructed from any CFG.
18	T	The set of Context Free Languages (CFL) is greater than the set of regular languages
19	T	Complement of any regular language is also a regular language
20	F	Complement of any non-regular language is also a regular language
21	T	PDA is more powerful than NFA
22	F	In PDA, we have a queue, where we enqueue and dequeue elements into/from that queue.
23	T	(0U1)* means everything over the alphabet $\{0,1\}$
24	F	If a language consists of only empty string, then this language is empty set.
25	F	There are finitely many regular languages.

(Solutions for the following questions can be found in the book or on the internet.)

2. Assume you are given the following Context Free Grammar (CFG):

$$\begin{aligned} R &\rightarrow XRX \mid S \\ S &\rightarrow \mathtt{a}T\mathtt{b} \mid \mathtt{b}T\mathtt{a} \\ T &\rightarrow XTX \mid X \mid \varepsilon \\ X &\rightarrow \mathtt{a} \mid \mathtt{b} \end{aligned}$$

a. What are the variables of G?
b. What are the terminals of G?
c. Which is the start variable of G?
d. Give three strings in $L(G)$.
e. Give three strings *not* in $L(G)$.
f. True or False: $T \Rightarrow \mathtt{aba}$.
g. True or False: $T \overset{*}{\Rightarrow} \mathtt{aba}$.
h. True or False: $T \Rightarrow T$.
i. True or False: $T \overset{*}{\Rightarrow} T$.
j. True or False: $XXX \overset{*}{\Rightarrow} \mathtt{aba}$.
k. True or False: $X \overset{*}{\Rightarrow} \mathtt{aba}$.
l. True or False: $T \overset{*}{\Rightarrow} XX$.
m. True or False: $T \overset{*}{\Rightarrow} XXX$.
n. True or False: $S \overset{*}{\Rightarrow} \varepsilon$.
o. Give a description in English of $L(G)$.

3. Assume you are given the following Context Free Grammar (CFG):

$$\begin{aligned} E &\rightarrow E + T \mid T \\ T &\rightarrow T \times F \mid F \\ F &\rightarrow (\, E\,) \mid \mathtt{a} \end{aligned}$$

Give parse trees and derivations for each string.

a. a
b. a+a
c. a+a+a
d. ((a))

4. Give the CFG for the following languages

a) {w | w contains exactly two 0s}
b) {w | w contains at least two 0s}
c) {w | w begins with a 1 and ends with a 0}
d) {w | w contains at least three 1s}

e) {w | the length of w is at most 5}
f) {w | w contains an even number of 0s, or contains exactly two 1s}
g) $\{0^n1^n \mid n>=0\} \cup \{1^n0^n \mid n>=0\}$

5. Give the Push Down Automata (PDA) for the following languages

a) {w | w contains exactly two 0s}
b) {w | w contains at least two 0s}
c) {w | w begins with a 1 and ends with a 0}
d) {w | w contains at least three 1s}
e) {w | the length of w is at most 5}
f) {w | w contains an even number of 0s, or contains exactly two 1s}
g) $\{0^n1^n \mid n>=0\} \cup \{1^n0^n \mid n>=0\}$